Large neutrino flavor mixings and gauge-mediated supersymmetry breaking scenario

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Abstract

The gauge-mediated supersymmetry breaking mechanism is one of the most reliable scenario which naturally suppresses the large flavor changing neutral current and CP violation in the supersymmetric standard model. When the messenger fields have suitable $B-L$ charges, the radiative correction naturally induces the Zee-like neutrino mass matrix, which provides tiny neutrino masses and large lepton flavor mixings. Our numerical results are consistent with the neutrino oscillation experiments in both three and four neutrino models.

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1 Introduction

Recent neutrino oscillation experiments provide a strong evidence of tiny neutrino masses and large lepton flavor mixings[1, 2, 3]. We know two mechanisms which can explain in a natural way the smallness of neutrino masses. One is the see-saw mechanism which can induce the small neutrino mass by integrating out heavy right-handed neutrinos[4]. The second scenario is that neutrinos obtain their masses by the radiative corrections through which the left-handed neutrinos obtain the Majorana masses. The latter is the so-called Zee model[5], which does not need right-handed neutrinos[6]. In this paper we consider the second scenario in the supersymmetric standard model.

We will need the suitable supersymmetry (SUSY) breaking mechanism in order to build a realistic SUSY theory. We consider for this purpose the low energy gauge-mediated SUSY breaking mechanism[7]. This is one of the most reliable scenario which can naturally suppress the large flavor changing neutral current and CP violation.

If there is no right-handed neutrino, neutrinos can not obtain the Dirac mass terms as in the standard model (SM). In order to obtain neutrino masses without right-handed neutrinos in the SUSY model the following three conditions are necessary:
(i) : $SU(2)_L$ must be broken,
(ii) : lepton number must be broken,
(iii) : supersymmetry must be broken.
Quarks and leptons can not obtain their masses without the first condition. The second condition is required to obtain Majorana neutrino masses. The lepton number conservation prevents neutrinos from obtaining the Majorana masses in the SM. Recall that neutrinos can obtain masses in the R-parity breaking scenario in the SUSY theory[9]. This is due to the fact that R-parity is broken whenever the lepton number is broken. The conditions (i) and (ii) are also needed in the see-saw mechanism which includes the right-handed neutrinos. The SUSY non-renormalization theorem requires the third condition (iii) since the neutrino masses are generated by the F-terms in the SUSY theory. If the SUSY is the exact symmetry, neutrinos can not obtain masses from the quantum corrections.

In this paper we propose a model where radiative corrections induce the tiny neutrino masses and large lepton flavor mixings. This is easily realized when in the three neutrino scenario in the SUSY theory the messenger fields have suitable $B-L$ charges, extra Higgs doublets, and two singlet fields which have lepton number. If three more extra singlet fields and one more pair of the messenger fields are added then four neutrino scenario is also realized which in particular explains LSND experiment[10].

2 Three neutrino model

Let us introduce the messenger model which can induce three tiny neutrino masses from the quantum corrections. We assume the gauge singlet field

$$\phi = \langle\phi\rangle + \langle F_\phi\rangle\,\theta^2 \tag{1}$$

in the SUSY breaking sector which couples the messenger fields. We introduce $\mathbf{10_M}+\overline{\mathbf{10_M}}$ messenger fields in the $SU(5)$ gauge representation. These components are given by

$$\mathbf{10_M} = (Q_M, \overline{U_M}, \overline{E_M}), \qquad \overline{\mathbf{10_M}} = (\overline{Q_M}, U_M, E_M), \tag{2}$$

with ordinary quantum charges for the SM gauge symmetry. They can mediate universal soft SUSY breaking parameters through ϕ by the flavor blind gauge interactions[7]*. The squark and slepton soft masses, and gaugino masses are given by $(\alpha/4\pi)(\langle F_\phi\rangle/\langle\phi\rangle)$ of order 10^2 GeV. On the other hand, scalar three point soft breaking terms (A-terms) are induced by the two-loop diagrams, which are estimated as $(\alpha/4\pi)^2(\langle F_\phi\rangle/\langle\phi\rangle) \sim 1$ GeV. Here α denotes gauge couplings.

The matter fields are given by

$$\mathbf{10_f} = (Q, \overline{U}, \overline{E}), \qquad \overline{\mathbf{5_f}} = (\overline{D}, L), \tag{3}$$

which are the same as the conventional $SU(5)$ grand unified gauge theory. The Higgs fields are given by

$$\Phi = (C, H), \qquad \overline{\Phi} = (\overline{C}, \overline{H}), \tag{4}$$

$$\Phi_e = (C_e, H_e), \qquad \overline{\Phi_e} = (\overline{C_e}, \overline{H_e}), \tag{5}$$

where triplets are colored Higgs $C, \overline{C}, C_e, \overline{C_e}$ which must be heavy enough to avoid rapid proton decay. H and $\overline{H}$ are the ordinary Higgs particles, and H_e and $\overline{H_e}$ are the extra Higgs doublets. We also introduce two gauge singlet fields χ and $\overline{\chi}$ which have the lepton number. The extra fields ($\mathbf{10_M}+\overline{\mathbf{10_M}}$) and ($\Phi_e+\overline{\Phi_e}$) have the conventional gauge quantum numbers, however they have unordinary $B-L$ charges Q_{B-L} as follows:

$$Q_{B-L} = Q_F + \frac{2}{5}Y, \tag{6}$$

where Y is the ordinary hypercharge and Q_F-charge is defined in Table 1.

*The universality of soft SUSY breaking parameters is modified, when there are messenger-matter mixings. This is because the Yukawa interactions can also mediate SUSY breaking parameters[8]. Actually, soft scalar masses in our following models are shifted by the effects of messenger-matter mixings. We will discuss the effects in summary.

Field	Φ	$\overline{\Phi}$	$\overline{\mathbf{5_f}}$	$\mathbf{10_f}$	$\mathbf{10_M}$	$\overline{\mathbf{10_M}}$	Φ_e	$\overline{\Phi_e}$	χ	$\overline{\chi}$
Q_F	$-\frac{2}{5}$	$\frac{2}{5}$	$-\frac{3}{5}$	$\frac{1}{5}$	$\frac{6}{5}$	$-\frac{6}{5}$	$\frac{8}{5}$	$-\frac{8}{5}$	2	-2
Z_2	$+$	$+$	$-$	$-$	$+$	$+$	$+$	$+$	$+$	$+$

Table 1: Q_F-charge in three neutrino model

We shall also introduce Z_2 symmetry which avoids matter-Higgs and matter-messenger field mixings†. The charge assignments in Table 1 determine the superpotential W_3 in three neutrino model given by

$$\begin{aligned} W_3 &= \mathbf{10_f}\mathbf{10_f}\Phi + \mathbf{10_f}\overline{\mathbf{5_f}}\,\overline{\Phi} + \mathbf{10_M}\overline{\mathbf{5_f}}\,\overline{\mathbf{5_f}} + \mathbf{10_M}\overline{\Phi}\,\overline{\Phi_e} + \overline{\mathbf{10_M}}\Phi\Phi_e \\ &\quad + \chi\Phi\overline{\Phi_e} + \overline{\chi}\Phi_e\overline{\Phi} + M\mathbf{10_M}\overline{\mathbf{10_M}} + \mu\Phi\overline{\Phi} + \mu_e\Phi_e\overline{\Phi_e} + \mu_\chi\chi\overline{\chi}, \end{aligned} \tag{7}$$

where M is the order of the messenger scale. The superpotential W preserves $U(1)_{B-L}$ global symmetry and Z_2 discrete symmetry. We assume that the triplet-doublet splitting is realized in the Higgs sector, where colored Higgs C's and $\overline{C}$'s have super-heavy masses, while Higgs doublets H's and $\overline{H}$'s have weak scale masses. Since the superpotential W_3 conserves $U(1)_{B-L}$ symmetry, proton-decay processes occur only when the diagrams contain dimension five operators $(QQQL)$ or $(\overline{D}\,\overline{D}\,\overline{D}\,\overline{E})$. Thus the proton-decay is suppressed enough in this model as in the ordinary grand unified models‡. We assume the Higgs fields $H, \overline{H}, H_e, \overline{H_e}$ and singlet fields $\chi, \overline{\chi}$ obtain vacuum expectation values of the weak scale order. Since $U(1)_{B-L}$ symmetry is spontaneously broken by the nonzeros of $\langle H_e\rangle$ and $\langle \overline{H_e}\rangle$ in this model, massless Majoron particles should appear. However, Majoron fields could almost decouple not only with quarks and leptons but also with gauge bosons because Majoron fields are almost composed by singlet fields in the appropriate parameter region. Therefore the existence of massless Majoron in this model is compatible with accelerator experiments.

Now let us estimate the neutrino masses and lepton flavor mixing angles. We extract the interaction of the lepton doublet in the third Yukawa coupling of Eq.(7), and denote it by $f_{\alpha\beta}\,\overline{E_M}L_\alpha L_\beta$, where indices correspond to the flavor as $\alpha, \beta = e, \mu, \tau$. Since the coupling $f_{\alpha\beta}$ is anti-symmetric tensor, diagonal elements of the neutrino mass matrix m_ν

$^\dagger Z_2$ symmetry is the extension of the conventional R-parity, which distinguishes not only the matter field $\overline{\mathbf{5_f}}$ (L) and the Higgs field $\overline{\Phi}$ $(\overline{H})$, but also the matter field $\mathbf{10_f}$ and the messenger field $\mathbf{10_M}$. Without Z_2 symmetry this model becomes R-parity breaking scenario, where neutrinos can obtain their masses by mixing with neutralinos.

‡The gauge coupling unification can not be realized in this model due to the existence of the extra light Higgs doublets. We must consider the unified theory with larger gauge group than $SU(5)$ and other extra fields at the high energy for the unification. What we can say here is that the proton decay through the colored Higgs exchange is suppressed enough because of their super-heavy masses as in the ordinary $SU(5)$ scenario.

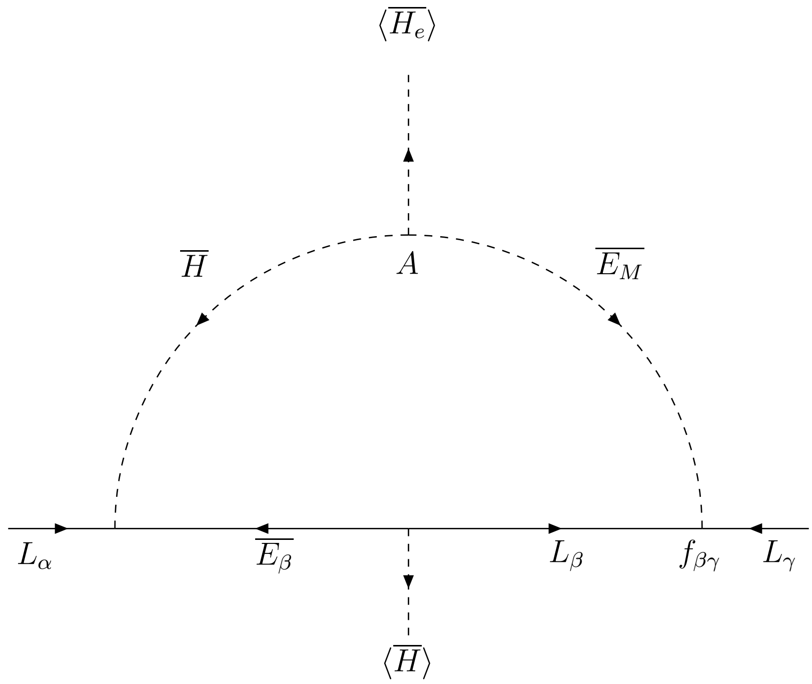

Figure 1: Feynman diagram to generate neutrino masses in the three neutrino model

become zero as follows:

$$m_\nu = \begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & 0 & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & 0 \end{pmatrix}. \tag{8}$$

This type of mass matrix leads to the stable lepton flavor mixing matrix[11] against the quantum corrections[12]. The diagram of Fig.1 indicates the neutrino masses as

$$\begin{aligned} m_{e\mu} &= f_{e\mu}(m_\mu^2 - m_e^2) A \frac{\langle \overline{H_e} \rangle}{\langle \overline{H} \rangle} F(M^2, \mu^2), \\ m_{e\tau} &= f_{e\tau}(m_\tau^2 - m_e^2) A \frac{\langle \overline{H_e} \rangle}{\langle \overline{H} \rangle} F(M^2, \mu^2), \\ m_{\mu\tau} &= f_{\mu\tau}(m_\tau^2 - m_\mu^2) A \frac{\langle \overline{H_e} \rangle}{\langle \overline{H} \rangle} F(M^2, \mu^2), \end{aligned} \tag{9}$$

where

$$F(M^2,\mu^2) = \frac{1}{16\pi^2}\frac{1}{M^2-\mu^2}\ln\frac{M^2}{\mu^2}. \tag{10}$$

Here A is the soft mass of the scalar three point coupling $\overline{E_M}\ \overline{H}\ \overline{H_e}$. When $f_{e\mu} \gg f_{e\tau} \gg f_{\mu\tau}$ and $f_{e\mu}/f_{e\tau} \simeq m_\tau^2/m_\mu^2$, the neutrino mass matrix m_ν in Eq.(8) can induce the bi-maximal mixings and suggest the atmospheric neutrino solution and solar vacuum solution. The unique mass matrix compatible with the solar and the atmospheric experiments in the Zee model with three neutrino requires (1,2) and (1,3) elements to be of the same order, and (2,3) element to be negligible when compared to (1,2) and (1,3) elements[13]. The bi-maximal condition $0.02 < m_{e\mu} < 0.08$ is realized, when $f_{e\mu} \sim 1$, $M \sim 10^{4.5}$ GeV[§], and $\langle\overline{H}\rangle < \langle\overline{H_e}\rangle$.

3 Four neutrino model

Next we present the messenger model which can induce four tiny neutrino masses by the quantum corrections. Let us introduce another $(\mathbf{10^0_M} + \overline{\mathbf{10^0_M}})$ messenger fields[¶] in the $SU(5)$ gauge representation, which are denoted by

$$\mathbf{10^0_M} = (Q^0_M, \overline{U^0_M}, \overline{E^0_M}), \qquad \overline{\mathbf{10^0_M}} = (\overline{Q^0_M}, U^0_M, E^0_M). \tag{11}$$

Since these fields have ordinary quantum charges for the SM gauge symmetry, they can mediate SUSY breaking through ϕ by the conventional gauge-mediated scenario. We also introduce two gauge singlet fields S, N, and $\overline{N}$ which have the lepton number. The extra fields $2(\mathbf{10}+\overline{\mathbf{10}})$, $(\Phi_e + \overline{\Phi_e})$, S, N, and $\overline{N}$ have Q_F charges as listed in Table 2. Here

Field	Φ	$\overline{\Phi}$	$\overline{\mathbf{5_f}}$	$\mathbf{10_f}$	$\mathbf{10_M}$	$\overline{\mathbf{10_M}}$	Φ_e	$\overline{\Phi_e}$	χ	$\overline{\chi}$	S	N	$\overline{N}$	$\mathbf{10^0_M}$	$\overline{\mathbf{10^0_M}}$
Q_F	$-\frac{2}{5}$	$\frac{2}{5}$	$-\frac{3}{5}$	$\frac{1}{5}$	$\frac{6}{5}$	$-\frac{6}{5}$	$\frac{8}{5}$	$-\frac{8}{5}$	2	-2	-1	-2	2	$-\frac{4}{5}$	$\frac{4}{5}$
Z_2	$+$	$+$	$-$	$-$	$+$	$+$	$+$	$+$	$+$	$+$	$-$	$+$	$+$	$+$	$+$
Z_3	1	1	1	1	1	1	1	1	1	1	ω	ω	ω^2	ω	ω^2

Table 2: Q_F-charge in four neutrino model

we also introduce Z_3 symmetry, which avoids the tree level mass of sterile neutrino S. The charge assignment in Table 2 determines the superpotential W_4 in the four neutrino

§The messenger scale of M and M_0 in Eq.(12) must be larger than 10^4 GeV. This lower bound is required by the positivity of the scalar masses of the messenger quarks and leptons. We thank Y. Mimura and Y. Nomura for this observation.

¶In this case, the gauge couplings blow up around 10^{14} GeV[14].

model, which is given by

$$W_4 \;=\; W_3 + S\mathbf{10_f}\overline{\mathbf{10_M^0}} + N\mathbf{10_M}\overline{\mathbf{10_M^0}} + M_0\mathbf{10_M^0}\overline{\mathbf{10_M^0}} + \mu_N N\overline{N}, \tag{12}$$

where W_3 is given by Eq.(7), and M_0 is of the order of the messenger scale. The superpotential W_4 preserves $Z_2 \times Z_3$ symmetry. In the four neutrino model we also assume that the triplet-doublet splitting is realized in the Higgs sector, and the Higgs fields $H, \overline{H}, H_e, \overline{H_e}$ and the singlet field N have vacuum expectation values of order of the weak scale.

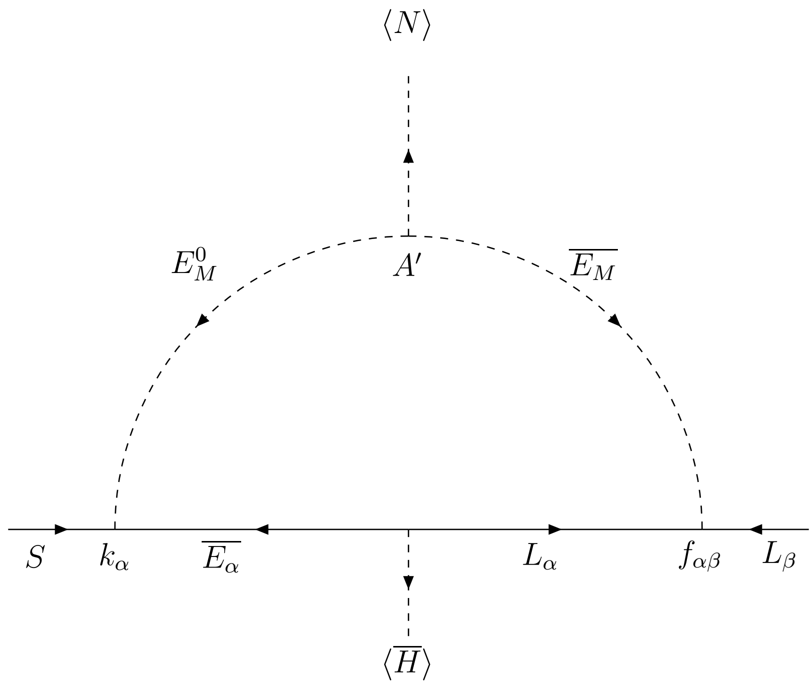

Figure 2: Feynman diagram to generate neutrino masses in the four neutrino model

Next we estimate the neutrino masses and lepton flavor mixing angles in the four neutrino model. We extract the interaction of the lepton by the second term in Eq.(12), and we denote it by $k_\alpha S\overline{E_\alpha}E_M^0$. The 4×4 neutrino mass matrix is given by

$$m_\nu = \begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} & m_{es} \\ m_{e\mu} & 0 & m_{\mu\tau} & m_{\mu s} \\ m_{e\tau} & m_{\mu\tau} & 0 & m_{\tau s} \\ m_{es} & m_{\mu s} & m_{\tau s} & 0 \end{pmatrix}. \tag{13}$$

The mass terms including the sterile neutrino field s[15] are obtained from the diagram of Fig.2 as

$$\begin{aligned} m_{es} &= (f_{e\tau}k_\tau m_\tau + f_{e\mu}k_\mu m_\mu)A'\langle N\rangle F(M^2, M_0^2), \\ m_{\mu s} &= (f_{\mu\tau}k_\tau m_\tau + f_{\mu e}k_e m_e)A'\langle N\rangle F(M^2, M_0^2), \\ m_{\tau s} &= (f_{\tau\mu}k_\mu m_\mu + f_{\tau e}k_e m_e)A'\langle N\rangle F(M^2, M_0^2), \end{aligned} \tag{14}$$

where A' is the soft mass of the scalar three point coupling $N\overline{E_M}E^0_M$, which is of order 1 GeV. The same type of mass matrix as in Eq.(13) was already analyzed in details in Ref.[16].

When $f_{e\mu} \ll f_{e\tau} < f_{\mu\tau}$ and $k_\tau \ll k_\mu \leq k_e$, this model can explain the LSND, the atmospheric and the solar neutrino experiments, in which the small mixing angle MSW solution is prefered. For example, when $f_{\mu\tau} \simeq 1$, $f_{e\tau} \sim 0.1$, $f_{e\mu} \simeq 10^{-3}$, $k_e = k_\mu \simeq 1$, and $k_\tau \simeq 10^{-3}$, the elements might be $m_{\mu\tau} = 0.5$ eV, $m_{e\tau} = 0.05$ eV, $m_{e\mu} = 10^{-5}$ eV, $m_{\tau s} = 0.15$ eV, $m_{\mu s} = 0.0036$ eV, and $m_{es} = 0.00025$ eV, which in turn, induce suitable neutrino mass squared differences $\delta m^2_{\rm sol} = 4\times10^{-6}$ eV^2, $\delta m^2_{\rm atm} = 2\times10^{-3}$ eV^2, $\delta m^2_{\rm LSND} = 0.3$ eV^2, and mixing angles $\sin^2 2\theta_{\rm sol} = 1\times10^{-3}$, $\sin^2 2\theta_{\rm atm} = 0.9$, $\sin^2 2\theta_{\rm LSND} = 0.03$[16].

4 Summary

Recent neutrino oscillation experiments suggest a strong evidence of tiny neutrino masses and large lepton flavor mixings. The scenario where neutrinos obtain their masses by radiative corrections without right-handed neutrino can explain the smallness of neutrino masses. On the other hand, the gauge-mediated supersymmetry breaking mechanism is one of the most reliable scenario which naturally suppresses the large flavor changing neutral current and CP violation in the SUSY theory.

In this paper we have shown the model which leads to the tiny neutrino masses with the maximal lepton flavor mixing. The Zee-like neutrino mass matrix is naturally realized in the three neutrino scenario in the frame of SUSY theory when the messenger fields have suitable $B-L$ charges, extra Higgs doublets, and two singlet fields, which have lepton number. If three more extra singlet fields and one more pair of the messenger fields are added then the four neutrino scenario is realized. This mass matrix is consistent with solar, atmospheric and LSND experiments with the appropriate values of parameters.

Finally, let us discuss the effects of messenger-matter mixings in our model. In three neutrino scenario, soft masses of the left-handed slepton doublets, the right-handed squarks, and Higgs fields are shifted as $\delta m^2/m^2 \simeq O(F_\phi^2/M^4)$. Since we do not know the coupling $f'_{\alpha\beta}$ of $Q_M L_\alpha \overline{D_\beta}$ which can be different from $f_{\alpha\beta}$ at the weak scale, we can not exactly estimate the shift of soft masses. The experiments of $K-\overline{K}$ and $\mu\to e\gamma$ require the constraints for the couplings of $f_{\alpha\beta}$ and $f'_{\alpha\beta}$. Naively, we can neglect the difference between the mass shift of the first generation and that of the second generation, since the Yukawa coupling $f_{\alpha\beta}$ is anti-symmetric and $f_{e\mu} \gg f_{e\tau} \gg f_{\mu\tau}$. In four neutrino model, soft masses of the right-handed slepton, the left-handed squark doublet, and right-handed up-type squark are also shifted. The FCNC experiments also give us the strong constraints for the couplings of messenger-matter mixings. A naive estimation shows us that the mass shift can be phenomenologically accepted since $k_e = k_\mu \simeq 1$, which induces the suitable neutrino mass matrix.

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